# NONLOCAL EFFECTS UNDER ELASTIC PERCOLATION CONDITIONS IN DEEP-LYING STRATA

V. N. Nikolaevskii

A nonlocal formulation is proposed for the hypothesis of the constancy of rock pressure for nonstationary percolation under pressure in a deep-lying elastic rockshelf. According to the formulation proposed, stress variations in the skeleton of the rockshelf are caused by changes in the interstitial pressure in the proximity of the area studied.

1. Nonstationary phenomena occurring during the percolation of homogeneous dropping liquid (petroleum or water) in deep-lying rockshelfs are associated with the effect of contraction of the interstitial space which accompanies a liquid-pressure drop. Contractibility is determined by hydrostatic expansion of the grains of the medium as well as by rock-skeleton compression under the action of rock pressure. Rigorous computation of transient processes must be based on a mathematical model of the fluid-saturated deformable porous medium [1, 2], with allowance for the stress redistributions which occur in the surrounding rock series. The corresponding boundary-value problem is, however, highly complex and, in particular, its formulation in each individual case is highly ambiguous due to the absence of detailed data on geological cross section and mechanical properties of the surrounding rocks. This is why extensive use is still made of the elementary theory of elastic percolation [2] based on the hypothesis of the constancy of rock pressure $\Gamma(x_i)$ at each point of a rockshelf of thickness 2h:

$$\sigma(x_1, x_2; t) + p(x_1, x_2; t) = \Gamma(x_1, x_2), \tag{1.1}$$

during transient processes, when the interstitial pressure p and the effective pressure $\sigma$ in the rock skeleton vary in time. A relation between porosity m and the pressures p and $\sigma$ is introduced, in a simplified statement, on the basis of experimental data. This makes it possible to reduce the continuity equation for the liquid phase and the motion equation (Darcy law)

$$\frac{\partial}{\partial t}(m\rho) + \operatorname{div}(\rho \mathbf{w}) = G, \qquad \mathbf{w} = -\frac{k}{\mu}\operatorname{grad} p, \tag{1.2}$$

in a linear approximation, to the following equation:

$$(a_\rho + a)\frac{\partial p}{\partial t} - b\frac{\partial \sigma}{\partial t} = \frac{k_0}{m_0\mu_0}\nabla^2 p + \frac{G}{m_0\rho_0}. \tag{1.3}$$

In this equation, there are introduced distributed sources and sinks $G(x_i, t)$ which simulate the effect of interstices through which the liquid penetrates into or is removed from the rock.

By using hypothesis (1.1), it is possible to further transform Eq. (1.3) to a piezoconductivity equation (the terminology is due to Shchelkachev [4])

$$\frac{\partial p}{\partial t} = \varkappa\nabla^2 p + q, \qquad \varkappa = \frac{k_0}{\mu_0\beta m_0}, \qquad q = \frac{G}{m_0\beta\rho_0}. \tag{1.4}$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 4, pp. 35-38, July-August, 1968. Original article submitted April 5, 1968.

Here, $\varkappa$ is the piezoconductivity coefficient; k is the permeability of the rock; $\beta$ is the compressibility of the rock; $\rho$ is the fluid density; $\mu$ is the viscosity of the fluid;

$$(\rho / \rho_0) = 1 + a_\rho (p - p_0); \quad (m / m_0) = 1 + a (p - p_0) - b (\sigma - \sigma_0);$$
$$\beta = a_\rho + a + b.$$

We note that for cemented sandstones

$$a \sim 5 \cdot 10^{-6} \text{ atm}^{-1}, \quad b \sim 10^{-4} \text{ atm}^{-1}, \quad m = 0.1 \div 0.2, \quad k = 10^{-10} \div 10^{-9} \text{ cm}^2,$$

for water

$$a_\rho \sim 5 \cdot 10^{-5} \text{ atm}^{-1},$$

for petroleum

$$a_\rho \sim 10^{-5} \div 10^{-3} \text{ atm}^{-1}.$$

The derivation of Eq. (1.4), first proposed by Jacob [3], was recently discussed in [5].

An analysis of different versions [3, 6, 7] of the formulation of the local hypothesis (1.1) is given in [8, 9]. In [9], it is shown in addition that neglect of the solid-particle displacement rate in the Darcy law, while simultaneously accounting for the compressibility of the solid phase in the continuity equation (1.2), is permissible for cemented porous media (the a/b ratio is equal to fractions of unity).

2. The local formulation of hypothesis (1.1) about the rock-pressure constancy at each point of the rock does not take into account that the surrounding shear-resistant rocks, with decreasing interstitial pressure, act not only as a load but as a ceiling as well. Indeed, if in a thin stratum (see figure) a change in interstitial pressure $\Delta p$ takes place only in a sufficiently narrow element (of length $l$), then, due to the work of the surrounding rocks as a ceiling (beam), changes in effective pressure in the middle of this element will not satisfy condition (1.1), i.e., $\Delta\sigma + \Delta\rho \neq 0$. Qualitatively, it can be seen that with increasing length $l$ the zone of interstitial pressure drop, the beam deflection, and, hence, the change $\Delta\sigma$ will increase. Here, there exists a certain characteristic length d, such that for $l \gg d$, the equality $\Delta\sigma + \Delta\rho = 0$ will be satisfied in the center of an isolated zone. The parameter d should therefore be considered as a quantitative characteristic of a given rockshelf (mechanical properties of the shelf itself and of the entire rock series).

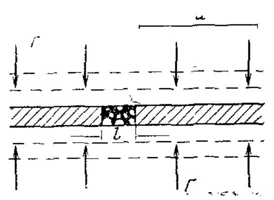
With the intention of retaining the elementary nature of the theory being developed, we formulate a nonlocal hypothesis about rock-pressure constancy, in the form

$$\sigma (x_i, t) + \iint \Phi (x_i, x_i'; d) p (x_i', t) dx_1' dx_2' = \Gamma (x_i). \tag{2.1}$$

Here, $\Phi (x_i, x_i'; d)$ is a certain influence function which depends parametrically on d, while integration is extended over the entire area of the rockshelf. For an isotropic homogeneous rockshelf, an approximation is permissible where the influence function is considered to depend solely on coordinate difference

$$\Phi (x_i, x_i'; d) = \Phi (x_i - x_i'; d)$$

(generally speaking, these conditions do not hold, because of the inhomogeneity and anisotropy of the interstitial pressure fields – in which case d is not a scalar).



Let us consider, for example, a function $\Phi$ of the form

$$\Phi \left( \frac{x_i - x_i'}{d} \right) = \frac{1}{\pi d^2} \exp \left\{ - \sum_{i=1,2} \frac{(x_i - x_i')^2}{d^2} \right\}. \tag{2.2}$$

Then, in the limiting case $d \to 0$, function (2.2) reduces to a delta function $\delta (x_1 - x_1') \, \delta (x_2 - x_2')$, while condition (2.1) degenerates to the local condition (1.1). In the other limiting case, $d \to \infty$, hypothesis (2.1) reduces to the condition $\partial\sigma/\partial t=0$, which implies that the mean stresses in the rock skeleton are constant in time. In this case, the pressure redistribution is described by Eq. (1.4), but with a large piezoconductivity coefficient that corresponds only to contractibility due to hydrostatic compression of the grain material of the medium and of the interstitial liquid.

It should be noted that by introducing a time dependence to the function $\Phi$, it becomes possible to take into account the creepage effects of the rock-series, while introduction of a time dependence to the resolving equation for porosity makes it possible to take into account the creepage effects of the rock skeleton itself.

The fact that the effective pressure in a rockshelf changes only when the mean rock pressure (i.e., a certain interstitial pressure averaged over the area) changes has been pointed out already by G. V. Isakov in 1948 [6]; however, an appropriate mathematical formulation for a postulation of this type is yet to be obtained.

3. If the influence function $\Phi$ depends only on the coordinate difference $x_i - x_i'$, then the problem can be solved with the aid of Fourier integrals [10, 11]. Indeed, by applying, for example, Fourier transforms to the Eqs. (1.3), (2.1), and (2.2), we get

$$(1-\alpha)\frac{dP}{dt} - \alpha\frac{d\Pi}{dt} + \varkappa(\xi^2+\eta^2)P = X(\xi, \eta, t),$$

$$\frac{d\Pi}{dt} = -F(\xi, \eta)\frac{dP}{dt}, \quad F(\xi, \eta) = \exp\left(-\frac{\xi^2+\eta^2}{4}d^2\right),$$

$$\Pi = L\sigma, \quad P = Lp, \quad X = Lq, \quad \alpha = \frac{b}{\beta},$$

$$Lf = \frac{1}{2\pi}\iint\limits_{-\infty}^{\infty} f(x_1, x_2; t)\, e^{i\xi x_1 + i\eta x_2}\, dx_1\, dx_2. \tag{3.1}$$

If $P_0 = Lp(x_i; t=0)$, then the general solution to (3.1) has the form

$$P = P_0(\xi, \eta)\, e^{-\varkappa t(\xi^2+\eta^2)A} + \int\limits_0^t X(\xi, \eta, \tau)\, A e^{-\varkappa(t-\tau)(\xi^2+\eta^2)A}\, d\tau,$$

$$A^{-1} = 1 - \alpha(1-F). \tag{3.2}$$

The desired solution for $p(x_i, t)$ is obtained by applying Fourier's converse theorem [11] to (1.3).

Let us examine, as an example, the problem of nonstationary pressure changes in a rockshelf into which at a moment of time $t=0$, a liquid mass $G_0$ was instantaneously introduced through a gallery located at cross section $x=0$. In this case

$$q(x_i, t) = q_0\delta(x_1)\delta(t)$$

and, hence,

$$X(\xi, \eta; \tau) = q_0\delta(\tau)/\sqrt{2\pi}.$$

Then, the solution has the form

$$p(x, t) - p_0 = \frac{q_0}{2\pi}\int\limits_{-\infty}^{\infty}\frac{e^{-i\xi x}}{1-\alpha(1-F(\xi))}\exp\left\{-\frac{\varkappa\xi^2 t}{1-\alpha(1-F(\xi))}\right\}d\xi = \frac{q_0}{\pi\sqrt{\varkappa t}}I,$$

$$I = \int\limits_0^{\infty}\frac{\cos mz}{1-\alpha[1-\exp(-\chi z^2/4)]}\exp\left\{-\frac{z^2}{1-\alpha[1-\exp(-\chi z^2/4)]}\right\}dz, \tag{3.3}$$

where use is made of the parity of the function's inverse transform desired and the notations

$$\chi = d^2/(\varkappa t),\ m = x/\sqrt{\varkappa t},$$

are introduced.

Given below are values of 1/2 I for $\alpha$ =1/2, obtained on a computer for various values of m and for $\chi$=0 and $\chi$=10:

| $m=0$ | 0.1 | 0.5 | 1 | 2 | 3 | |
|---|---|---|---|---|---|---|
| $0.5I=0.4431$ | 0.4420 | 0.4163 | 0.3451 | 0.1630 | 0.0467 | $(\chi=0)$ |
| $0.5I=0.4386$ | 0.4380 | 0.4208 | 0.3706 | 0.2147 | 0.0662 | $(\chi=10)$ |

From (3.3) derive solutions for limiting special cases ($\chi\to 0$ and $\chi\to\infty$), which can be interpreted as asymptotic solutions

$$p(x,t)-p_0=\frac{q_0}{\sqrt{\varkappa t}}\frac{\sqrt{\pi}}{4}\exp\left(-\frac{x^2}{4\varkappa t}\right),\quad d^2\ll\varkappa t,\quad x\sim\sqrt{\varkappa t}\quad(\chi\to 0); \tag{3.4}$$

$$p(x,t)-p_0=\frac{q_0(1-\alpha)^{-1}}{\sqrt{\varkappa_1 t}}\frac{\sqrt{\pi}}{4}\exp\left(-\frac{x^2}{4\varkappa_1 t}\right),\quad \varkappa_1=\frac{\varkappa}{1-\alpha},$$
$$d^2\gg\varkappa t,\quad x\sim\sqrt{\varkappa t}\quad(\chi\to\infty). \tag{3.5}$$

As expected, in motion regions much larger than measure d, the solution conforms roughly with ordinary local theory. If the motion region is much smaller than measure d, local theory is also suitable as an approximation; however, the effective piezoconductivity coefficient is then larger $\varkappa_1=\varkappa\,(1-\alpha)^{-1}$. In this case, for the same amount of admitted liquid, the pressure must be higher than that obtained from local theory (compare the values computed for m$\sim$1, $\chi$=10).

The author is indebted to É. A. Avakyan and N. A. Efremova for performing the computations.

## LITERATURE CITED

1. Ya. I. Frenkel, "Contribution to the theory of seismic and seismoelectric phenomena in humid soil," Izv. AN SSSR, ser. geogr. i geofiz., vol. 8, no. 4, 1944.
2. V. N. Nikolaevskii, "Basic dynamic equations of liquid-saturated elastic porous media," collection: Petroleum Extraction [in Russian], Nedra, Moscow, 1964.
3. C. E. Jacob, "On the flow of water in an elastic artesian aquifer," Tran. Amer. Geophys. Union, Reps, and Papers, part 2, Hydrology, Washington, Nat. Acad. Sci., D.C., 1940.
4. V. N. Shchelkachev, "Basic motion equations for an elastic fluid in an elastic medium," Dokl. AN SSSR, vol. 52, no. 2, 1946.
5. R. G. M. De Wiest, "On the storage coefficient and the equations of groundwater flow," G. Geophys. Res., vol. 71, no. 4, 1966.
6. G. V. Isakov, "Deformations of petroleum collectors," Neft. Khoz-vo, no. 11, 1948.
7. G. I. Barenblatt and A. P. Krylov, "Elastic-plastic percolation regime," Izv. AN SSSR, OTN, no. 2, 1955.
8. J. Geertsma, "The effect of fluid pressure decline on volumetric changes of porous rocks," J. Petr. Technol., vol. 9, no. 12, 1957.
9. P. P. Zolotarev and V. N. Nikolaevskii, "Expansion of pressure waves in liquid-saturated rocks," Tr. Vses. neftegaz. n-i. in-ta, no. 42, pp. 112-130, 1965.
10. E. C. Titchmarsh, Introduction to the Theory of Fourier Integrals [Russion translation], Gostekhizdat, Moscow-Leningrad, 1948.
11. I. N. Sneddon, Fourier Transforms [Russian translation], Izd-vo inostr. lit., Moscow, 1955.